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Abstract

The article intents to show that correction formula for gas-filled detectors has to contain the half-life constant of specimen, i.e. that characteristic of specimen may have influence on measurement in some rear situations. There is given a more accurate formula for correction of dead-time interval for gas-filled detectors and its deriving procedure.

DEAD TIME CORRECTION OF NON PARALYZING X RAY DETECTORS

In the following text the next symbol's definitions are adopted:

- $\mathbf{t_p}$ - detector's time of paralyzing, i.e. dead time interval,
- λ - constant of radioactive decay for nucleus, $\lambda = \mathbf{Ln(2) / T_{1/2}}$,
- $\mathbf{N_0}$ - the initial number of non-decayed nucleuses,
- **N** - the number of decayed nucleuses,
- **M** - the number of detected decays,
- **k** - paralyzing factor.

The estimation of non detected particle's decays is based on the simple relation of radioactive decay that compute the number of decayed nucleuses for the dead time of detector paralysis $\mathbf{t_p}$:

$$N = N_0 \cdot \left(1 - e^{-\lambda \cdot t_p}\right) \tag{1}$$

The value of paralyzing coefficient **k** is obtained by dividing the left and the right side of the above equation with the number of non-decayed nucleuses for the time of paralysis:

$$k = 1 - e^{-\lambda \cdot t_p} \tag{2}$$

Now it is possible to derive the difference equation of non-paralyzing detector:

$$N(M) = N(M-1) + 1 + \left(N_0 - \left(N(M-1) + 1\right)\right) \cdot k \tag{3}$$

This equation has to have the next initial condition:

$$N(0) = 0 \tag{4}$$

The variable **M** in difference equation is the number of **M**-th detected decay. On the left side of difference equation is the number of really decayed nucleuses. On the right side is the number of really decayed nucleus before **M**-th decay, plus a just detected one, plus the estimation of non-detected nucleuses that have come on the just detected decay. It is necessary to be known the number of non-decayed nucleuses after detection of **M**-th decay while the esteem of non-detected decays is obtained via multiplication of coefficient **k** with the number of non-decayed nucleuses because the paralysis begins after detection of that decay after that there is certainly a non-decayed nucleus less.

The solution of that difference equation is:

$$N(M) = \frac{\left(1-(1-k)^{M}\right)\cdot\left(1+k\cdot(N_0-1)\right)}{k} \qquad (5)$$

After substituting the value for **k** in previous equation, it is obtained:

$$N(M) = \frac{\left(1-e^{-M\cdot\lambda\cdot t_p}\right)\cdot\left(1+\left(1-e^{-\lambda\cdot t_p}\right)\cdot(N_0-1)\right)}{1-e^{-\lambda\cdot t_p}} \qquad (6)$$

Suppose that the speeds of counting and decay are known $\left(\frac{\mathbf{dN}}{\mathbf{dt}} \text{ and } \frac{\mathbf{dM}}{\mathbf{dt}} \text{ respectively}\right)$, then the next formula is valid:

$$\frac{dN(M)}{dt} = \frac{dN(M)}{dN}\cdot\frac{dM}{dt} = \frac{\lambda\cdot t_p\cdot e^{-\lambda\cdot t_p\cdot M}\cdot\left(1+\left(1-e^{-\lambda\cdot t_p}\right)\cdot(N_0-1)\right)}{1-e^{-\lambda\cdot t_p}}\cdot\frac{dM}{dt} \qquad (7)$$

There is also a valid formula:

$$N = N_0 - \frac{1}{\lambda}\cdot\frac{dN}{dt} \qquad (8)$$

$\frac{\mathbf{dN}}{\mathbf{dt}}$ is directly derived by combining the last three formulas:

$$\boxed{\frac{dN}{dt} = \frac{\lambda\cdot t_p}{e^{\lambda\cdot t_p}-1}\cdot\frac{\frac{dM}{dt}}{1-t_p\cdot\frac{dM}{dt}}} \qquad (9)$$

If we assume that are $\mathbf{n} = \frac{\mathbf{dN}}{\mathbf{dt}}$ and $\mathbf{m} = \frac{\mathbf{dM}}{\mathbf{dt}}$, than is obtained:

$$\boxed{n = \frac{\lambda\cdot t_p}{e^{\lambda\cdot t_p}-1}\cdot\frac{m}{1-t_p\cdot m} = \left(\frac{t_p}{T_{1/2}}\cdot\frac{Ln(2)}{2^{\frac{t_p}{T_{1/2}}}-1}\right)\cdot\frac{m}{1-t_p\cdot m} \approx \left(1-\frac{\lambda\cdot t_p}{2}\right)\cdot\frac{m}{1-t_p\cdot m}} \qquad (10)$$

The relation between λ and $\mathbf{T_{1/2}}$ is given by next formula:

$$\lambda = \frac{Ln(2)}{T_{1/2}} \qquad (11)$$

The classic correction formula for non-paralyzing detectors could be simply derived from this formula by introduction of the linear approximation of the exponential function (**$e^x \approx 1 + x$**).

Absolute value of relative error of classic formula (13) related on the formula (9) is given by the next expression:

$$\left|\frac{\Delta n}{n}\right| = \frac{e^{\lambda \cdot t_p} - 1}{\lambda \cdot t_p} - 1 \approx \frac{\lambda \cdot t_p}{2} + \frac{(\lambda \cdot t_p)^2}{6} \qquad (12)$$

The formula (9) shows that half-life constant of specimen may have influence on measurement performed with the gas-filled detector especially when the decay's chain of specimen contains extremely short-living isotopes with the similar order of magnitude as the dead-time constant of detector.

The classic formula

$$\frac{dN}{dt} = \frac{\frac{dM}{dt}}{1 - t_p \cdot \frac{dM}{dt}} \qquad (13)$$

does not include the specimen half-life constant at all. If we assume that is:

$$x = \frac{t_p}{T_{1/2}} \qquad (14)$$

Then the fist part of equation (10) that involves influence of decay constant of specimen to counting result will be transformed to:

$$f(x) = \frac{x \cdot Ln(2)}{2^x - 1} \qquad (15)$$

The following table contains correction values for some characteristic ratios between **t_p** and **$T_{1/2}$**:

$\frac{t_p}{T_{1/2}}$	$f\left(\frac{t_p}{T_{1/2}}\right)$
0	1
¼	0.915857710411652423343
½	0.83670266201424625056
1	0.69314718055994530941
2	0.46209812037329687294

REFERENCES

1. G. F. Knoll, *Radiation Detection And Measuremen*t, (John Wiley & Sons, Inc., 2nd edition 1989).
2. N. Tsoulfanidis, *Measurement And Detection Of Radiatio*n, (McGraw Hill Book Company, 1983).