GENERAL THRUSTER'S EQUATIONS

ABSTRACT

This article shows that propulsion's drive can be entirely descried by equations (13), (15), (21), (22) and (25).
These equations can lead us to construction of much better and efficient jet engines and also some other engines based on ion propulsion, MHD, etc.

Basic definition of linear momentum of a jet propulsion is:

$$\vec{P} = m \cdot \vec{v} \tag{1}$$

Whereas:

$\vec{v}$ = velocity of jet in regards to the thruster device,
m = mass of the jet,
$\vec{P}$ = momentum, pulse.

Force of blowing jet is defined as derivative of pulse per time:

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{dm}{dt} \cdot \vec{v} = \dot{m} \cdot \vec{v} \tag{2}$$

Dot over variable denotes time derivation. In the case it is a mass of air per time. Kinetic energy of the jet is defined as:

$$E = m \cdot \frac{\vec{v}^2}{2} \tag{3}$$

Where E is kinetic energy of the blowing jet.
Power of thruster device is defined as time derivative of energy:

$$W = \frac{dE}{dt} = \frac{dm}{dt} \cdot \frac{\vec{v}^2}{2} = \dot{m} \cdot \frac{\vec{v}^2}{2} \tag{4}$$

A view to equations (2) and (4) shows us that force of thruster is proportional to velocity and that consumed power is proportional to square of velocity. The intention of any reactive power design except for rocket motors is to blow as much air as slower as possible. This is reason why helicopters have big propeller that is rotating slowly – to push as much air as slower as possible.
Relation between density ρ and mass m is given by the following equation:

$$m = \rho \cdot V \tag{5}$$

Whereas V is volume of substance. Thus we have:

$$V = \frac{m}{\rho} \tag{6}$$

We can differentiate above equation on time and we have:

$$\dot{V} = \frac{\dot{m}}{\rho} \tag{7}$$

In this case ρ is density of the air. It can be approximated with 1kg/m^3 (precisely 1.293kg/m^3).
Basic definition of velocity is:

$$\vec{v} = \frac{d\vec{\ell}}{dt} \tag{8}$$

We can evolve above equation:

$$\vec{v} = \frac{d\vec{\ell}}{dt} \cdot \frac{\vec{S}}{\vec{S}} = \frac{1}{\vec{S}} \cdot \frac{dV}{dt} = \frac{\dot{V}}{\vec{S}} \tag{9}$$

Whereas $\vec{S}$ is surface of the blowing aperture of the jet motor or thruster. Usually velocity of the jet and hole's surface vectors are collinear, thus vector signs could be omitted. This is not case for vector jet engines, but for most thrusters we can suppose that they blow air collinearly with their axe of symmetry.
If we introduce (7) into (9) we have that velocity of the jet is:

$$\vec{v} = \frac{1}{\vec{S}} \cdot \frac{\dot{m}}{\rho} = \frac{\hat{S}}{\left|\vec{S}\right|} \cdot \frac{\dot{m}}{\rho} \tag{10}$$

We can replace the velocity in equation (2) and than we have:

$$\vec{F} = \dot{m} \cdot \vec{v} = \frac{1}{\vec{S}} \cdot \frac{\dot{m}^2}{\rho} \tag{11}$$

This equation shows that force of a thruster is equal to blowing mass per time divided by the area of hole and density of the fluid, in our case, density of the air.
Power W of the engine is obtained by introduction of (10) into (4):

$$W = \frac{\dot{m}}{2} \cdot \vec{v}^2 = \frac{\dot{m}}{2} \cdot \left(\frac{1}{\vec{S}} \cdot \frac{\dot{m}}{\rho} \right)^2 = \frac{\dot{m}^3}{2 \cdot \vec{S}^2 \cdot \rho^2} \tag{12}$$

Relation between force and power is derived from equations (11) and (12), from (12) we have:

$$\dot{m} = \sqrt[3]{2 \cdot W \cdot \vec{S}^2 \cdot \rho^2} \tag{13}$$

It could be replaced in (11):

$$\vec{F} = \dot{m} \cdot \vec{v} = \frac{1}{\vec{S}} \cdot \frac{\left(2 \cdot W \cdot \vec{S}^2 \cdot \rho^2\right)^{\frac{2}{3}}}{\rho} = \sqrt[3]{4 \cdot W^2 \cdot \left|\vec{S}\right| \cdot \rho} \cdot \hat{S} \tag{14}$$

For collinear thruster with perfect efficiency:

$$F = \sqrt[3]{4 \cdot W^2 \cdot S \cdot \rho} \tag{15}$$

For real thruster the formula has an additional variable, efficiency η of the engine, $\eta \in [0,1)$:

$$\vec{F} = \sqrt[3]{4 \cdot \eta^2 \cdot W^2 \cdot \left|\vec{S}\right| \cdot \rho} \cdot \hat{S} \tag{16}$$

Because:

$$W_{eff} = \eta \cdot W \tag{17}$$

For most engines:

$$F = \sqrt[3]{4 \cdot \eta^2 \cdot W^2 \cdot S \cdot \rho} \tag{18}$$

Force or thrust (F) is expressing in Newtons or kilograms ([N]=9.81·[kg]) power (W) is expressing in watts or horse powers, surface (S) is expressing in square meters (m^2), density of fluid (ρ) is expressing in kilograms per cubic meter (kg/m^3) and efficiency (η) is number between 0 and 1. Efficiency defines how much energy is not transferred into curled movement of fluid, heating of fluid or sound energy. Higher value means higher efficiency and better engine.
Velocity of the jet can be computed by arranging of (4) and (13):

$$W = \sqrt[3]{2 \cdot W \cdot \vec{S}^2 \cdot \rho^2} \cdot \frac{\vec{v}^2}{2} \tag{19}$$

$\Rightarrow$

$$v^2 = \frac{2 \cdot W}{\sqrt[3]{2 \cdot W \cdot S^2 \cdot \rho^2}} = \sqrt[3]{\frac{4 \cdot W^2}{S^2 \cdot \rho^2}} \tag{20}$$

Velocity of jet is:

$$v = \sqrt[3]{\frac{2 \cdot W}{S \cdot \rho}} \tag{21}$$

From (7) and (13) we have that volume per time of jet is:

$$\dot{V} = \sqrt[3]{\frac{2 \cdot W \cdot S^2}{\rho}} \tag{22}$$

Pressure on the jet hole is simply computing from the following equation:

$$W = P \cdot \dot{V} \tag{23}$$

Where P is pressure. From (22) and (23) we have:

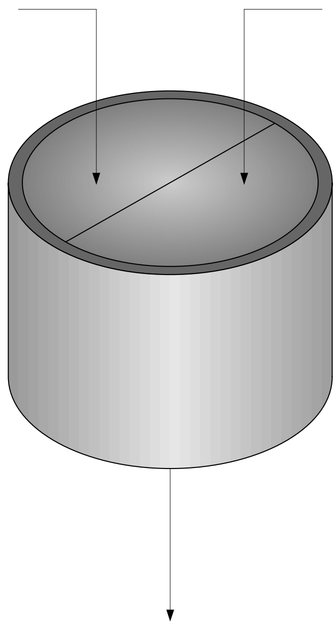
$$\frac{W}{P} = \sqrt[3]{\frac{2 \cdot W \cdot S^2}{\rho}} \tag{24}$$

Pressure is:

$$P = \sqrt[3]{\frac{\rho \cdot W^2}{2 \cdot S^2}} \tag{25}$$

Equations (13), (15), (21), (22) and (25) represent mass flow, force, velocity of the jet, volume per time of the jet and pressure on the blowing hole of the thruster respectively. These equations completely describe behavior of thrusters on small velocities of vehicles.
This equation should be verified on the fact that it should be additive one. On the following picture is showing one thruster that is divided on two equal surfaces:

Fig. 1



Fictive partition of the thruster on two equal areas with half of power each should not affect the total thrust force.
This is described mathematically on the following way:

$$F(2 \cdot S, 2 \cdot W) = 2 \cdot F(S, W) \tag{26}$$

When this is replaced into (18) we have:

$$\sqrt[3]{4 \cdot \eta^2 \cdot (2 \cdot W)^2 \cdot (2 \cdot S) \cdot \rho} = 2 \cdot \sqrt[3]{4 \cdot \eta^2 \cdot W^2 \cdot S \cdot \rho} \tag{27}$$

$\Rightarrow$

$$\sqrt[3]{32 \cdot \eta^2 \cdot W^2 \cdot S \cdot \rho} = \sqrt[3]{32 \cdot \eta^2 \cdot W^2 \cdot S \cdot \rho} \tag{28}$$

This is correct. This is proof that the formula is correct in infinitesimal sense and that every point could be reached with infinite dividing of a region.
We can compute power of thruster for required thrust using equation (18):

$$W = \frac{F^{\frac{3}{2}}}{2 \cdot \sqrt{\rho \cdot S}} = \frac{1}{2} \cdot \sqrt{\frac{F^3}{\rho \cdot S}} \tag{29}$$

For a thruster which power is defined as consumed (not as effective blowing power) we have:

$$W = \frac{F^{\frac{3}{2}}}{2 \cdot \eta \cdot \sqrt{\rho \cdot S}} = \frac{1}{2 \cdot \eta} \cdot \sqrt{\frac{F^3}{\rho \cdot S}} \tag{30}$$

Let us compute force of a 100% efficiency thruster device with blowing surface of 1m^2 and with 400W of power in regards with equation (15):

$$F(400W) = 86.17N \approx 8.78kg \tag{31}$$

If we decrease the surface of aperture on 0.5m^2 than we have:

$$F = 68.39N \approx 6.97kg \tag{32}$$

If we suppose that efficiency of the thruster is 95% have 6 and 8 kilograms of thrust. This is far away than ordinary propellers. Ordinary vacuum clearer has efficiency of only 20% for best models. The situation is similar with all propeller airplanes. Jet airplanes have better efficiency, about 30%. Prop-fan jet engines have efficiency of nearly 40%.
One two kilowatts on quarter of square meter (diameter of rounded aperture is 56cm) will produce thrust:

$$F = \sqrt[3]{4 \cdot 0.95^2 \cdot (2000W)^2 \cdot 0.25m^2 \cdot 1.293 \frac{kg}{m^3}} = 167.12N \approx 17Kg \tag{33}$$

But, with half of square meter (diameter of rounded hole is 80cm) we have:

$$F = 210.56N \approx 21.5kg \quad (34)$$

Let us compute what is the power needed to lift a man with a lifer device (about 150Kg of weight):

$$W = \frac{F^{\frac{3}{2}}}{2 \cdot \eta \cdot \sqrt{\rho \cdot S}} = \frac{\left(150kg \cdot 9.81\frac{N}{kg}\right)^{\frac{3}{2}}}{2 \cdot 0.95 \cdot \sqrt{1.293\frac{kg}{m^3} \cdot 0.5m^2}} = 36948.98W \approx 37KW \quad (35)$$

Lifter's power equation is simple; it is product of potential and current:

$$W = U \cdot I \quad (36)$$

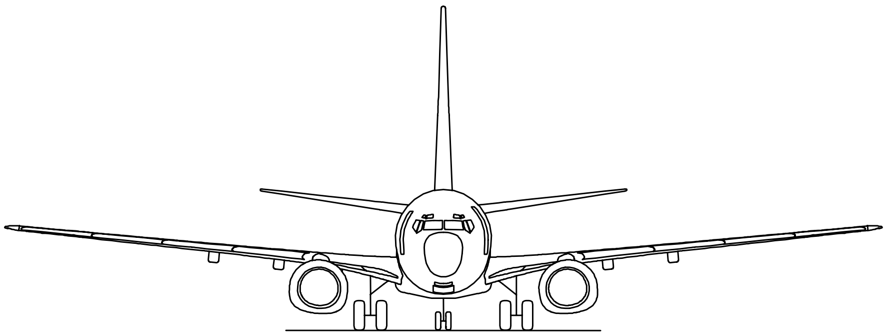
Generally we want to increase thrust and to decrease power. Equations (15) and (18) tell us that we should increase surface of the blowing pipe to decrease energy consumption, i.e. power. We cannot affect to density of the air because it is parameter of atmosphere and altitude. We can notice that with altitude air becomes spare and density decreases that additionally diminish thrust. However, area of motors pipe and density of air are linearly connected with the force, and efficiency and power are in square connection with power and thus their influence is much bigger then surface and density. Thus increase of the surface can affect the energy consumption a lot.
So, the difference between lifter, helicopter's propeller and jet engine is only in efficiency. I suppose that lifter has the best efficiency because there is no vortex flow and there is no heating of the jet. The vortex is presented in both cases of helicopters and jet engines caused by rotation of propeller and turbine respectively.
Russians tried to minimize the effect with two rotors on same axe (all Kamov's helicopters like Ka-226 and Ka-32a), i.e. two propellers on some models of their helicopters but with the approach they have increased the friction between propellers and air, which minimized gain. However these helicopters have highest altitude and they are using for rescuing alpinists on highest mountains.
Thus, equation (18) tells us that we want to maximize force and minimize power, i.e. energy consumption of the thruster device. Only way is to increase the blowing surface and to decrease the potential because velocity of the jet is proportional to the potential.
Equation (15) can be applied to slant foil with the angle θ between foil and horizontal axe of airplane and than we have that minimal power of engine to keep airplane in air is:

$$W = \sqrt{\frac{(m \cdot g)^3}{4 \cdot g \cdot S \cdot COS(\theta)}} \quad (37)$$

Whereas W is power of motor, m is mass of airplane, S is area of foils, g is gravity acceleration and θ is angle between foil and horizontal axe.

New airplanes have so big front side (hole) of jet engines that they cannot be circular, they are rounded from the nether side lest to catch the runway:

Fig. 2



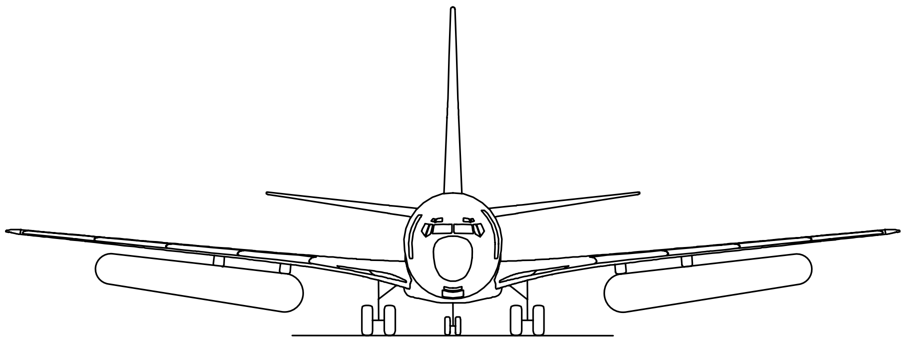
Boeing 737

Equation (18) is different than the Tsialkovsky equation because strategy of rocket engine design is diametrically different than thrusters' design. Rockets are caring repelling with themselves and thrusters are feeding from the environment. So, in case of rocket the goal is to accelerate repelling jet as fast as possible regardless the energetic efficiency because the amount of the repelling fluid is limited by the rocket's reservoir.
The NASA was using the Tsialkovsky approach designing the "Deep Space" ion rocket motor repelling on heavy ions and very high potential obtained from the small nuclear reactor able to supply the device nearly with unlimited amount of energy.
Lifter is feeding with the fluid from environment like every other propeller of jet engines, thus Sikorsky approach should be applied.

IMPROVEMENT

The way of improving of airplanes and decrease fuel consumption is usage of motors with no moving parts. By applying of improved statoreactor’s design or lifters we can build motors with big sucking holes’ areas due to absence of rotating moving parts in those engines thus the shape of the holes is not necessary circular. Such motor can be settled below the wings lest caching runway:

Fig. 3

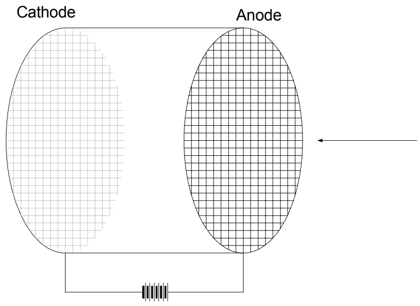


Boeing 737 with lifters

In additional analysis of lifter operation it is clearly shown that we should keep potential as low as possible. This can be achieved by heating of above wire or increasing of density of blowing elements, etc.

Lifter is invented by Thomas Townsend Brown in the year of 1923 and basic principle is shown on the following picture:

Fig. 4

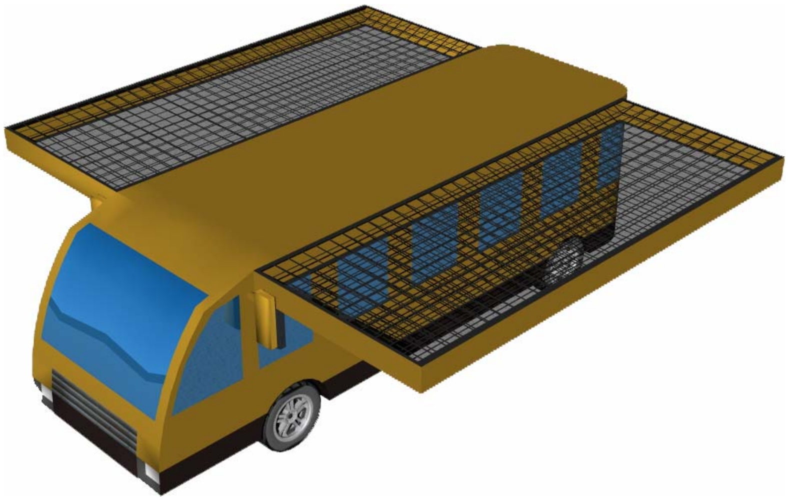


There is ions' wind between electrodes driven with high voltage and the thrust is:

$$F = \sqrt[3]{4 \cdot \eta^2 \cdot U^2 \cdot I^2 \cdot S \cdot \rho} \quad (38)$$

Advantage of lifter over all other thrusters is that it does not have moving and rotating parts and thus it has highest efficiency because there is no vortex component of the jet.
A bus with the blowing lattices with the doubled surface of bus ceiling area can lift the whole bas with the power obtained from the ordinary bus motor only:

Fig. 5



Further intensive analysis that includes velocity and compression of the air is not part of this document and all ones interested in a more detailed document should contact author on the following address:

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